



# Time-dependent spatial distribution of thermal stresses in the ice cover of a small reservoir



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## ABSTRACT

Static ice loads (ice actions) are a key design parameter for dams in cold climates. However, their theoretical description is still elusive, introducing uncertainty in design and hindering development of remediation measures. We present and analyze measurements of stresses due to thermal loads in a small reservoir in northern Norway. Several weeks of observations, including both cold and warm spells, were well-described by a simple equation that accounts for thermal expansion and temperature-dependent creep. One model parameter was found to depend systematically on the location of measurements within the reservoir. Biaxial stress measurements showed that the stress field was not homogeneous. Results suggest that the stress field in reservoirs should be predictable from first principles with numerical methods and point toward a promising, simple parameterization.

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## 1. Introduction

Design rules and practices have been established in several countries to ensure the safety of hydropower dams. One design parameter is static ice load, resulting from thermal expansion of the ice cover or water level fluctuations (e.g. Comfort et al., 2003). Ice loads are variable actions whose intensity and/or points of application vary frequently and significantly over time (CFBR, 2013). Design standards are based on limited empirical measurements as current understanding of the magnitude of ice loads is still limited (Comfort et al., 2003; Gebre et al., 2013; Timco et al., 1996). Since field measurements of ice forces on dams have suggested that maximum static ice loads depend significantly on the location of the dam, climate and sudden temperature changes, increasing efforts to understand ice loads on dams has been called for (Gebre et al., 2013). Benefits could be two-fold: on the one hand, costs may be reduced during dam construction or maintenance. On the other hand, solutions for ice load reductions may be developed: a more profound understanding of the mechanisms involved in ice-structure interactions may lead to the development of more cost efficient strategies for reducing the actual ice-load on dams. This may be of particular interest for the design of dams for small-scale hydropower plants. The potential for cost reductions during the lifetime of dams is exemplified by a recent change in the Norwegian regulation to the Water Resources Act (Vannressursloven) with respect to safety at hydropower plants, implemented in 2010 (Ministry of Petroleum and Energy, 2009). The guidelines and design recommendations include

new considerations regarding ice loads on dams (NVE, 2003), that cause a need for upgrading and strengthening of existing dams and infrastructure.

According to the Norwegian guidelines (NVE, 2003), thermal ice loads are assumed to be line loads between 100 and 150 kN/m near the top of the dam. However, during the most recent decades this has been verified in only one measurement campaign during a single season at Silvann dam in the northern part of Norway (Hoseth and Fransson, 1999).

This study focuses on thermal ice loads. Measurements were performed in a reservoir at a time of year when the ice level was kept steady by the balance of a small influx of water from a creek and outflow over the spillway beneath the ice over.

## 2. Methods

### 2.1. Installation

A weather station and stress cells were installed at Taraldsvikfossen Reservoir near Narvik, 68.4405° N, 17.471° E, 212 m above sea level in December 2012. The reservoir is small (1000 m<sup>3</sup>), bound at the western side by a straight concrete gravity dam and extending approximately 30 m in North–South direction (Fig. 1). A concrete maintenance hut is situated half way along the dam. Water depth in the reservoir is approximately 6 m at the dam, reducing to 1 m at the eastern shore. The Taraldsvik creek enters the reservoir from the North-East. A weather station was mounted at the maintenance hut, and a 3 m long vertical string of temperature sensors was frozen into the ice to measure the temperature profile through water, ice, snow, and air.

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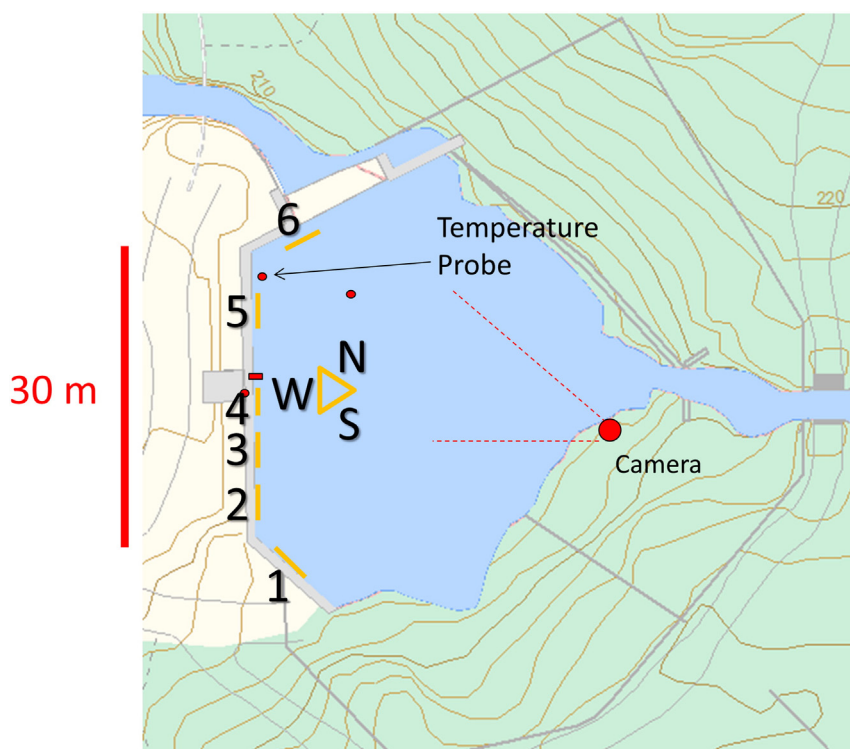


**Fig. 1.** Sketch of the approximate locations of the load cells deployed on 12 February 2013. Markers indicate station name. North is up. Map source: NVE Atlas.

As summarized by Cox and Johnson (1983), sensors used for ice stress measurements fall into two categories: cylindrical sensors having an effective modulus much greater than ice, and thin, wide sensors (flat-jacks), preferably having an effective modulus close to that of ice. Flat-jacks and cylindrical stress gauges were found to compare well during a recent series of field measurements in ice-covered reservoirs in Canada (Morse et al., 2011; Taras et al., 2011). In this study, stresses in the ice were monitored with 15 custom designed oil-filled GeoKon 4850 stress cells (essentially the same as those used by Carter et al. (1998) and Taras et al. (2011)). The cells consisted of two rectangular steel plates (100 mm × 200 mm) welded together around the periphery with de-aired oil occupying the space between the plates. A short tube

connected the cell to a vibrating wire pressure transducer that also measured temperature with a temperature-dependent resistor. The cells were calibrated by the manufacturer to an accuracy <0.5 kPa. Stress cells were mounted on steel tape at desired vertical separations and attached to wooden support (Figs. 2 and 3). In each case the center of the upper-most stress cell was 0.2 m below the ice surface at deployment. Two stations along the dam had three stress cells spaced 0.15 m (Stations 3 and 5) while the station toward the center of the reservoir measuring stress normal to the dam (Station W) had cells spaced 0.25 m. All instruments were connected to a CR1000 data logger recording stress data every 5 min. A timelapse camera overlooked the NW corner of the dam, including part of the spillway. The camera recorded images

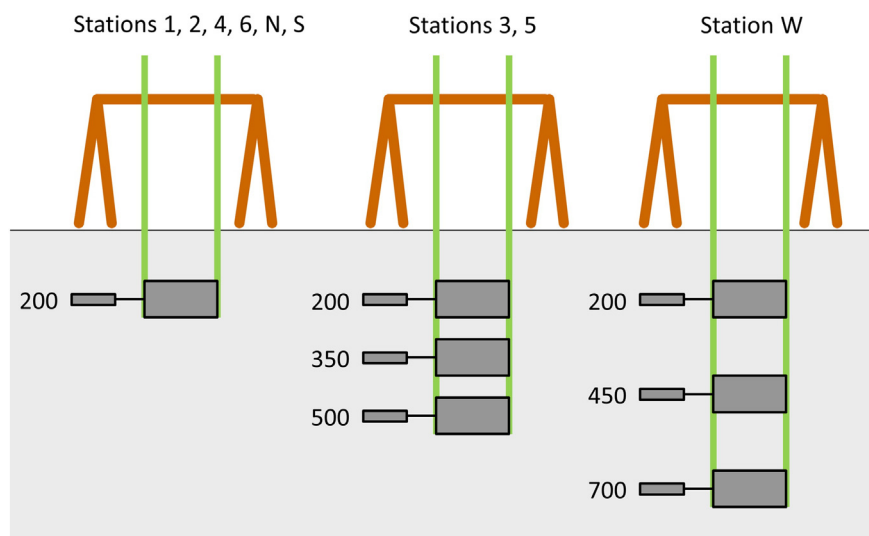


**Fig. 2.** Configuration of pressure cells (large rectangles) at 9 measurement stations. Depths of the centers of the vertical cells are given in mm below ice surface at the time of deployment.



Fig. 3. Installation of stress cells at Station 5 on 12 February 2013.

at an interval of approximately 30 min. A wooden snow depth stake marked in 10 cm intervals was frozen into the ice in view of the camera. Snow depth, freeboard, and ice thickness were measured at 1 cm resolution during site visits throughout the measurement season. Stress cells were deployed on 12 February 2013 (Fig. 3). Slots were cut with a chain saw, instruments were deployed attached to wooden support, and the slots were filled with water pumped from the reservoir. With the exception of Stations N and S, all pressure cells were mounted to register stresses in direction normal to the dam. Stations 1 to 6 were placed approximately 1 m in front of the dam into the ice with Stations 1 and 6 located on the flanks rather than the main section of the dam (Fig. 1, Table 1). Stations N, S, and W were installed approximately

Table 1

Position of stress cells. Lateral position is measured from the center of dam, except Stations 1 and 6 which are measured from the respective corners of the dam. Negative numbers are to the south. Normal position is measured perpendicular to the dam face.

| Station     | 1    | 2     | 3    | 4    | 5   | 6   | N   | S   | W   |
|-------------|------|-------|------|------|-----|-----|-----|-----|-----|
| # of cells  | 1    | 3     | 1    | 3    | 3   | 1   | 1   | 1   | 3   |
| Lateral (m) | −4.2 | −10.3 | −5.9 | −0.3 | 9.4 | 6.4 | 0.7 | 0.7 | 0.7 |
| Normal (m)  | 1.2  | 1.1   | 1.1  | 1.1  | 1.4 | 1.8 | 7.8 | 7.8 | 7.8 |

10 m in front of the dam and rotated 120° with respect to each other in an equilateral triangle with 2 m side length to allow principal stresses to be calculated easily (Holister, 1967).

## 2.2. Stress data processing

By convention, the zero-point of each stress cell was the respective reading of the pressure sensor after immersion into water. Changes of atmospheric pressure were not accounted for in data processing (order of 1 kPa). The following sign convention is used: stresses are positive in compression. In a confined isothermal plate, compressive stresses arise during temperature increase.

Principal stresses were calculated for the triangular configuration of stations N, S, and W following Holister (1967). The angle of the first principal stress is measured counter-clockwise from the surface normal of the dam.

## 2.3. Stress model

Thermal stresses in ice have been discussed in the context of elastic behavior over short periods of time, only (e.g. Comfort et al., 2003; Morse et al., 2011). However, ice is clearly not an elastic material and alternative formulations of ice rheology have been proposed (e.g., Bergdahl, 1978; Drouin and Michel, 1974; Lindgren, 1970; Royen, 1922). The model of Bergdahl (1978) enjoyed considerable success when compared with field measurements when the originally proposed coefficients were adjusted (e.g., Cox, 1984; Fransson, 1988). In addition, with only three parameters, this formulation is one of the simplest. Following Bergdahl (1978), the thermal stress in ice is described as a spring and non-linear dashpot model that can be expressed as

$$\frac{d\sigma}{dt} = E \left[ \alpha \frac{dT}{dt} - KD \left( \frac{\sigma}{\sigma_0} \right)^n \right], \quad (1)$$

where  $\sigma$  is the local stress (Pa),  $t$  is time (s),  $E$  the elastic modulus of ice (Pa),  $\alpha$  the thermal expansion of ice ( $\alpha = 5 \times 10^{-5} \text{ K}^{-1}$ ),  $T$  the ice temperature (°C),  $D$  the temperature-dependent viscous creep rate ( $\text{m}^2/\text{s}$ ),  $K$  ( $\text{m}^{-2}$ ) and  $n$  coefficients of viscous deformation, and  $\sigma_0$  an arbitrary reference stress for dimensional consistency (Pa). The temperature-dependent elastic modulus of ice is

$$E = E_0(1 - CT), \quad (2)$$

where  $C = 0.012 \text{ K}^{-1}$  (Bergdahl, 1978). While Bergdahl (1978) used  $E_0 = 6.1 \text{ GPa}$ , Cox (1984) suggested the use of  $E_0 = 4 \text{ GPa}$  since strain rates are low (Traetteberg et al., 1975). Bergdahl's choice of the Arrhenius equation to describe the temperature-dependence of the creep rate was questioned by Cox (1984), arguing that this relationship does not hold at high temperatures. Instead, based on very limited data, Cox (1984) suggested replacing  $KD$  with an expression

$$KD = \beta \left( \frac{T_1}{T} \right)^m, \quad (3)$$

where  $T_1 = -1^\circ \text{C}$  is a constant for dimensional scaling, and  $\beta = 2.46 \times 10^{-29} \text{ s}^{-1}$  and  $m = 1.92$  are fitted constants.

Fransson (1988) allowed all parameters to be fitted to data and, to avoid the singularity at  $0^\circ \text{C}$ , suggested

$$KD \propto \left( \frac{T_1}{T_1 + T} \right)^m. \quad (4)$$

Following Bergdahl and Wernersson (1978), Cox (1984) chose  $n = 3.7$ .

In this study stress was modeled as

$$\frac{d\sigma}{dt} = A \frac{dT}{dt} - B \left( \frac{T_1}{T} \right)^m \left( \frac{\sigma}{\sigma_0} \right)^n, \quad (5)$$

where  $A$ ,  $B$ , and  $n$  are temperature-independent constants. Constants  $m$  and  $T_1$  are as given by Cox (1984). However, three alternative formulations will be discussed, i.e.,

$$\frac{d\sigma}{dt} = A \frac{dT}{dt} - B \left( \frac{T_1}{T_1 - T} \right)^m \left( \frac{\sigma}{\sigma_0} \right)^n, \quad (6)$$

based on (4),

$$\frac{d\sigma}{dt} = (1 - CT) \left[ A \frac{dT}{dt} - B \left( \frac{T_1}{T} \right)^m \left( \frac{\sigma}{\sigma_0} \right)^n \right], \quad (7)$$

accounting for the temperature dependence of the elastic modulus (2), and

$$\frac{d\sigma}{dt} = A \frac{dT}{dt} - B \left( \frac{\sigma}{\sigma_0} \right)^n, \quad (8)$$

assuming temperature-independence of all physical properties.

While formulations (5) to (8) are defined for positive stresses, only, the formulation has been extended to negative stresses presumably the same way Cox (1984) did. This approach assumes that tensile stresses do not lead to crack formation or other irreversible deformation. Using (5) as an example, the model was implemented as

$$\frac{d\sigma}{dt} = A \frac{dT}{dt} - \text{sign}(\sigma) B \left( \frac{T_1}{T} \right)^m \left( \frac{|\sigma|}{\sigma_0} \right)^n, \quad (9)$$

where the  $\text{sign}(x)$  function evaluates to  $+1$  for  $x > 0$ ,  $-1$  for  $x < 0$  and  $0$  for  $x = 0$ .

The model equations were solved using explicit time integration since ice temperature and stress data were recorded at intervals as short as 5 min. Parameters  $A$ ,  $B$ , and  $n$  were fitted by minimizing the residual with a simplex algorithm (Nelder and Mead, 1965). In cases where the optimal common parameters for a group of stress cells have been determined (e.g., a common  $A$  and  $n$  for a group of cells while each cell differs in  $B$ ), the sum of the residuals of the individual cells was minimized.

We chose as reference case Eq. (9) with  $A = 200 \text{ kPa}/^\circ\text{C}$  and  $n = 3.7$  (following Cox (1984)) and  $B$  fitted to measurements. Unless specified otherwise, parameters were fitted to measurements of the combined period of 9 to 19 March and 26 March to 4 April as these were periods dominated by compression and spanned a wide range of ice temperatures.

### 3. Results

#### 3.1. Ice conditions

As common for reservoirs, the ice cover grew as congelation ice, snow-ice and superimposed ice from flooding (e.g., Carter et al., 1998; Leppäranta, 2015; Morse et al., 2011; Stander, 2006). At the time of deployment, ice thickness near the dam and at the center of reservoir was 0.75 m and 1.2 m, respectively, with 0.2 to 0.32 m snow depth at all stations except Station 5. Winds sweeping from the east had reduced snow depth to 0.05 m at Station 5.

Temperature data showed that the water was stratified until a warm spell occurred at the end of February (Petrich et al., 2014). Throughout March and April water was mixed presumably due to a steady flow of water moving through the reservoir. This flow across the spillway prevented water level fluctuations and the build-up of pressure in the reservoir (Petrich et al., 2014).

Ice conditions in Season 2012/13 were nearly ideal for the measurement of thermally-induced stresses. For convenience of the discussion, the measurement period discussed here was divided into several phases (Table 2). When stress cells froze into the ice cover in mid-February high initial stresses were recorded (reaching up to 1 MPa) due to volume expansion during freezing of surrounding water (e.g. Carter et al., 1998; Cox, 1984). Stress readings returned to their original zero-reading during a warm spell from 24 February until 1 March in all near-surface cells (i.e., 1, 2, 3-top, 4, 5-top, 6, W-top, N, and S). During the subsequent temperature decrease (phase 1a), none of those cells showed freeze-in artifacts of magnitude or duration observed in mid-February. This resulted in approximately six weeks of quality data of thermal stresses until the ice finally decayed starting 14 April (phase 6). The warm spell in February resulted in approximately 0.05 m ice ablation at the surface, i.e. the centers of the cells discussed here were at a depth of 0.15 m in the ice.

Episodes of snowfall ensured that the ice was continuously snow-covered from 4 Mar until 16/17 April with snow depth 0.1 m or more at the measurement stations (Fig. 4). The exception was phase 3 which started with wind reducing the snow cover to as little as 0.05 m at Station 5 on 19 March. Hence, radiative heating of the ice and cells was not of major concern. There was a small (3 m<sup>2</sup>), localized flooding event confined a patch of ice between Stations 5 and 6 on 8 March (followed by snowfall on 9 March, phase 2a), and a more extended event near the NW and SW corners of the reservoir (approx. 100 m<sup>2</sup> each) on 20 March (followed by snowfall on 22 March, phase 3). These locations were observed to experience flooding during cold spells throughout the winter (i.e., also before February) and coincided with the locations of a known crack (cf. Stander, 2006). Neither flooding reached a measurement station.

#### 3.2. Temperature registered by stress cells

Fig. 5 shows temperatures registered by the stress cells. Temperatures registered by stress cells 2, 3-top, 5-top, and 6 were very similar to each other (spread  $< 0.2^\circ\text{C}$ ) while temperatures at cells W-top, N, and S were systematically lower (up to  $0.5^\circ\text{C}$ ). Temperatures at cells 1 and 4 were higher and showed less variability than any of the other cells, resulting from deeper snow cover locally. Based on these observations, a temperature representative of the near-surface conditions of the ice sheet was calculated as arithmetic mean of cells 2, 3-top, 6, W-top, N, and S. Temperatures of cell 5-top were not used because the temperature signal was noisy, presumably due to a bad electrical contact. The representative temperature was used for ice stress modeling.

After an initial drop of the representative ice temperature to  $-9^\circ\text{C}$  in phase 1a in response to a decrease in air temperature, ice temperatures

**Table 2**

Division of the measurement period into phases based on stresses at cells 1, 2, 3-top, 4, 5-top, and 6; general stress state (compression/tension), distribution (evenly/uneven with pronounced peak), location of peak stress with respect to center of the dam (north or south), and general near-surface ice temperature.

| Phase | Date                      | State             | Distribution | Peak stress | Ice temp. |
|-------|---------------------------|-------------------|--------------|-------------|-----------|
| –     | Until 1 March (noon)      | No stress         |              |             | High      |
| 1a    | 1 to 5 March (am)         | Tension           | Uneven       | N           |           |
| 1b    | 5 to 7 March (am)         | Compression       |              |             |           |
| 2a    | 7 to 9 March (am)         | Compression       | Even         | S           |           |
| 2b    | 9 to 19 March (pm)        | Compression       | Even         | S           |           |
| 3     | 19 to 23 March (midnight) | Tension           | Uneven       | N           |           |
| 4a    | 23 to 31 March (pm)       | Compression       | Even         | S           |           |
| 4b    | 31 March to 5 April (am)  | Compression       | Even         | S           | High      |
| 5a    | 5 April to 8 April (noon) | Tension           | Uneven       | N           |           |
| 5b    | 8 April to 12 April (am)  | Stresses around 0 |              |             |           |
| 5c    | 12 April to 14 April (pm) | Tension           | Uneven       | N           |           |
| 6     | 14 April to 17 April (am) | Compression       | Even         | S           | High      |
| –     | From 17 April             | No stress         |              |             | High      |



Fig. 4. Overview of snow-covered site on 23 March 2013. View is toward north.

increased with periodic oscillations until the beginning of phase 5 with the exception of a cold spell in phase 3 ( $-8^{\circ}\text{C}$ ). The low ice temperatures in phase 3 were due to winds reducing the depth of the snow cover and increasing turbulent heat transfer. Diurnal temperature variations became evident in March in phase 4a, and persistent ice temperatures above  $-1^{\circ}\text{C}$  were observed in phase 4b. Ice temperatures stayed between  $-2$  and  $-1^{\circ}\text{C}$  throughout phase 5. Phase 6 marks the onset of ice decay with ice temperature increasing to the melting point.

### 3.3. Normal stresses close to the dam

Stresses measured along the dam spanned the range from  $-100$  kPa (tension) to  $+200$  kPa (compression). The ability of sensors to register tension was limited by a combination of cracks opening in the ice over, finite adhesion of the sensor to the ice, and theoretically expected boiling of the oil inside the cells at pressures below  $-100$  kPa. Tension has been observed during phases 1a, 3, and 5a (Fig. 6), each of which coincided with decreasing ice temperatures (Fig. 5).

Fig. 7 shows examples of horizontal profiles of normal stresses recorded by the surface-most cells in front of the dam. Prior to the end of March, highest stresses were observed at the center of the dam. Around the turn

from March to April, stresses were highest south of the center, while ice at the center tended to be in tension in early April. There were periods with relatively evenly distributed stresses (e.g. 1 April) and periods with distinct localized stress maxima (e.g. 25 March).

Time series of individual sensors are shown in Fig. 6. Initial cooling took place during phase 1a, followed by another brief cold spell in phase 1b. Compressive stress oscillated with a period of 4 stress maxima over the course of 14 days in phase 2. A cold spell in the ice had been experienced during phase 3. The ice warmed up continuously during phase 4 with a small superimposed diurnal temperature and stress signal. Stress magnitudes were small during phase 5, showing both tension and compression. Ice temperatures increased to the melting point in phase 6. The highest stresses were recorded at the end of the cold spells, i.e. in phase 1b and at the end of phase 3.

Using  $A$  and  $n$  as in Cox (1984), parameter  $B$  of the model of Eq. (9) was fitted to the data. The model was driven with the representative ice temperature for the depth of the cell rather than the locally observed stress cell temperature. The difference is illustrated in Fig. 9 which shows that the stresses measured during phase 4a were much better correlated with the representative ice temperature than with the local stress cell temperature. This difference most likely resulted from a

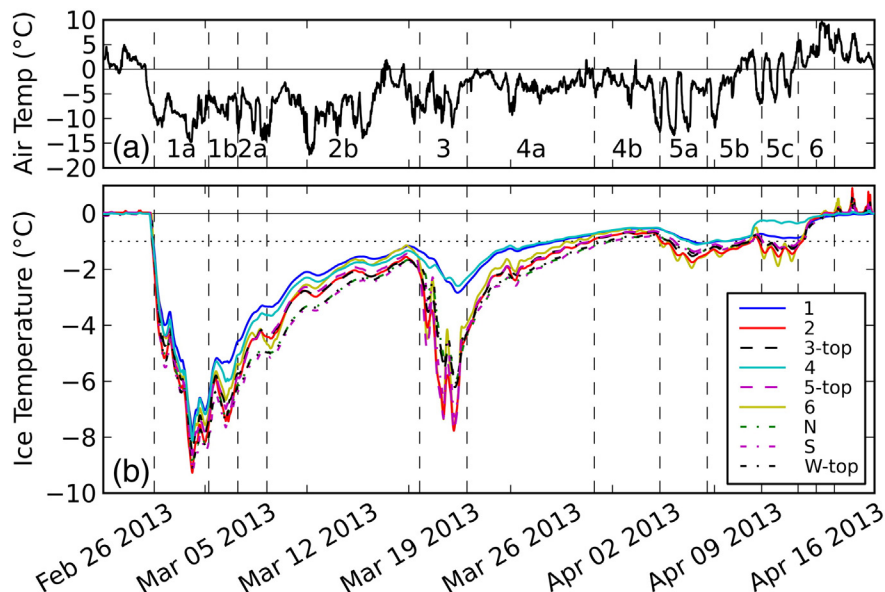


Fig. 5. (a) Air temperature and (b) temperatures registered at the top-most cell of each station. The dotted line at  $-1^{\circ}\text{C}$  is for visual reference to particularly warm ice.

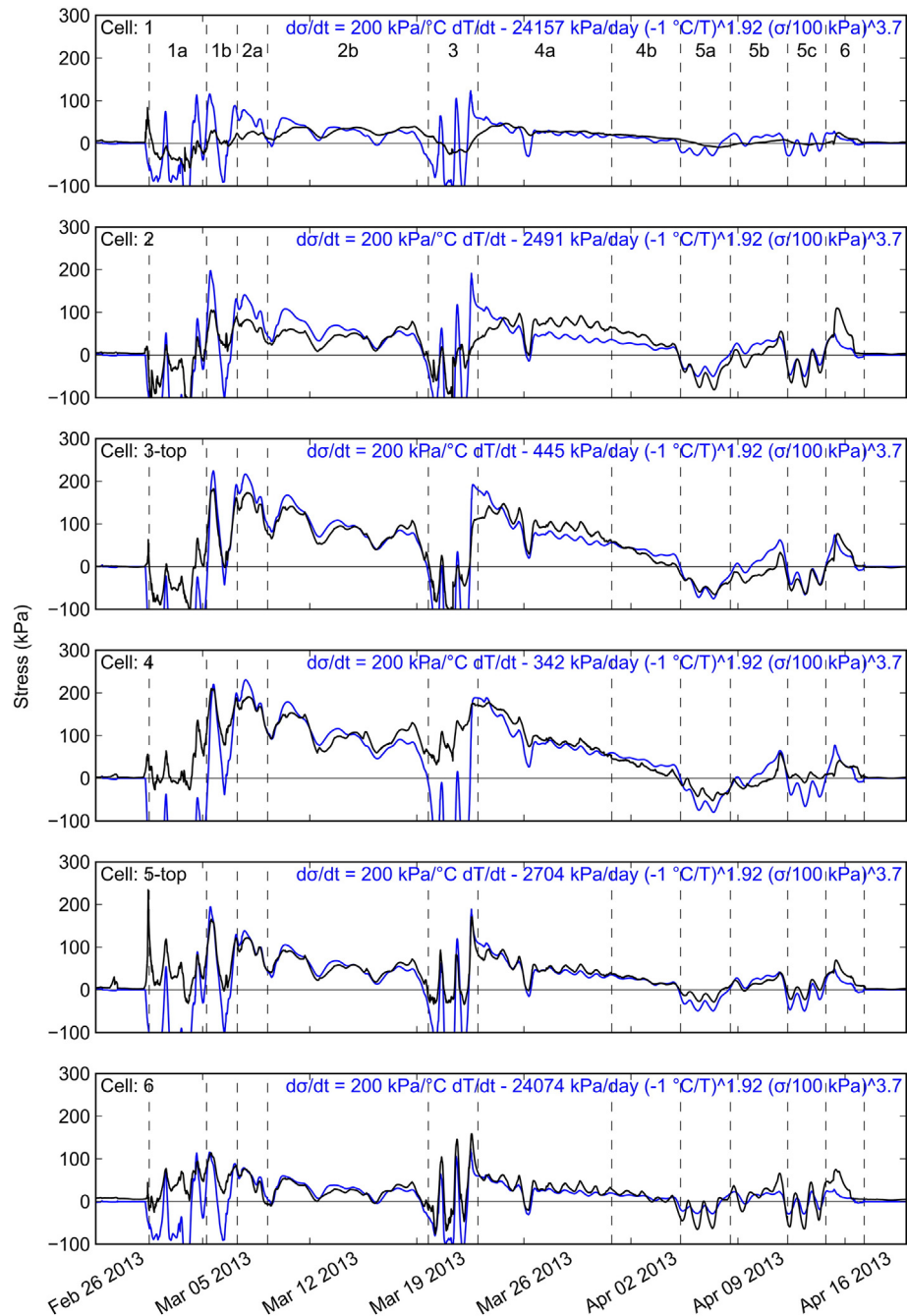


**Fig. 6.** Stresses registered in the near-surface cells along the dam (black lines). All simulations use average temperature registered at stations with thin snow cover (blue lines). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

thermally insulating snowdrift in front of the maintenance hut, extending to Station 4.

Fig. 6 compares model predictions with measured stresses. In spite of its simplicity, and although only phases 2b, 4a since 26 March, and 4b were used for fitting, the model results agree very well with observations throughout the season.

The fitted factor  $B$  shows systematic behavior, decreasing from the flanks of the reservoir (Stations 1 and 6,  $B = 24000$  kPa/day) toward the center (Station 4,  $B = 340$  kPa/day) (Fig. 8).  $B$  appears to change by a factor of approximately 10 over the course of 10 m. We are not aware of other reports of a systematic, spatial variation of coefficients in a reservoir. Our fitted values  $B$  are large compared to those of Cox (1984), who used  $B = 27$  kPa/day in a setting that was laterally

essentially unconstrained. Hence, in addition to creep (which is spatially independent),  $B$  also includes (spatially dependent) effects due to spatially varying boundary conditions, crack pattern, and possibly ice thickness and texture.

#### 3.4. Principal stresses

The principal stress direction varied systematically (by approximately  $90^\circ$ ) at the beginning and end of each cold spell lasting for about five days. Fig. 10 shows the principal stresses and the stress data of cells N, S, and W-top they were derived from. Periods of temperature increase (phases 2, 4, 5b, and 6) were associated with a counter-clockwise rotation of the principal stresses. Cold spells (phases 1a, 3,

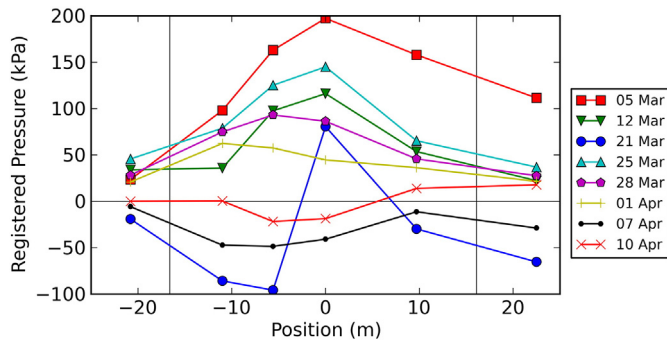


**Fig. 7.** Examples of pressure distribution at the top-most cells of stations 1 to 6. Data of midnight UTC, except 12:00 on 5 Mar 2013. The vertical lines mark the limits of the main section of the dam (i.e., S–N section). The x-axis shows linear distance from the center of the dam with negative values south (i.e., Stations 1 to 3).

5a, and 5c) started with a clockwise rotation of typically  $90^\circ$  that aligned the first principal stress in NW–SE direction. It is interesting to note that the first principle stress is generally aligned parallel to the dam. This appears to be consistent with other observations (e.g., Cox, 1984; Morse et al., 2011). Morse et al. (2011) attributed this to the presence of cracks parallel to the dam that harbor the potential to relieve stress. However, in the present case it could be related to the boundary conditions of the reservoir.

#### 4. Discussion

While the model does an excellent job reproducing the stress history of the reservoir, in particular during times of compression, it still relies on a single, location-dependent parameter that, so far, needs to be fitted to data. However, of most concern to design are maximum loads. In some cases the model predicted peak magnitudes that were not observed in cells. If one assumes that this is due to natural inhomogeneity of the ice then otherwise accurate models may need to include calibration factors to down-correct average loads on dams. Another possibility that should be considered is that measurements are at error, for example due to local changes in ice properties during the freeze-in period of the cell.

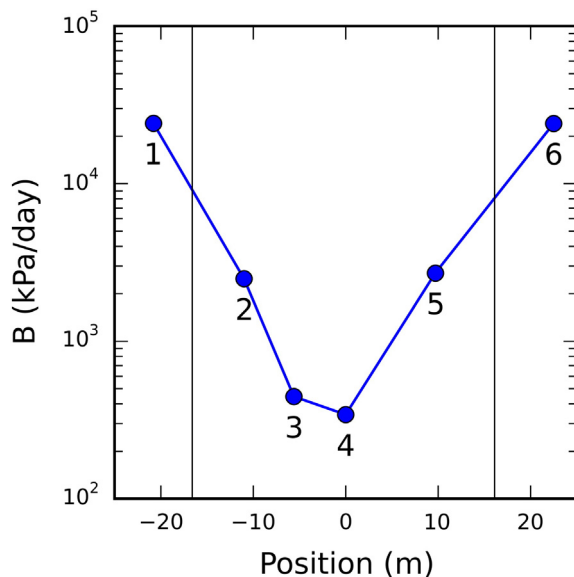


**Fig. 8.** Distribution of fitted parameter  $B$  (Fig. 6) along the stations (labels). The vertical lines mark the limits of the main section of the dam (i.e., S–N section). The x-axis shows linear distance from the center of the dam with negative values south (i.e., Stations 1 to 3).

#### 4.1. Normal stresses

The model of Bergdahl (1978) has been shown to be successful for stresses normal to a dam or shore. Ice stresses were investigated in one of the smallest reservoirs ever reported, and a systematic dependence of one parameter on location was found. It is obvious that the model in its current form cannot describe the biaxial stress field without additional considerations (e.g., Fransson, 1988). At the same time, this failure is reassuring us that the stresses measured were indeed brought about by the entire ice sheet rather than generated locally at the stress cells. However, it also serves as a reminder that Eq. (5) is first and foremost an empirical model, and that its applicability appears to be limited to stresses normal to and close to a rigid boundary.

A striking observation of the fitted parameters of Eq. (5) is that parameter  $B$  depends systematically on the position of the cell. It was lowest at the center of the dam where the sensitivity of load to temperature changes was highest and decreased toward either end of the dam (Fig. 8). Based on the introduction of Eq. (5) one would have expected  $B$  to be an intrinsic material property of ice only. However,  $B$  depends obviously on measurement location with respect to the boundary configuration. In addition,  $B$  may depend on the crack pattern and possibly systematic variations in ice thickness and texture. Both would result from the reservoir shape (cracks as a result of stresses, and thickness variations as a result of flooding through cracks), making it impossible to determine the physical origin from the current data set. A general procedure to determine the apparent parameters  $A$ ,  $B$ , and  $n$  (e.g., from rigorous numerical modeling) would greatly facilitate stress calculations.

#### 4.2. Model parameters

We noticed that it is possible to obtain reasonable fits using the same parameters  $A$  and  $n$  for all cells, only determining  $B$  individually for each cell. In contrast, it is not possible to do the same with common  $B$  and  $n$ , varying  $A$ . A lot of time can be spent on finding optimal parameters as the best fit depends somewhat on the time interval used for fitting. In the current case, the mathematically most optimal values for  $A$  and  $n$  common to all cells were determined to be  $A = 179 \text{ kPa}/^\circ\text{C}$  and  $n = 4.5$ . Forcing  $n = 3.7$  and  $n = 3$ , the optimal value would be  $A = 156 \text{ kPa}/^\circ\text{C}$  and  $A = 131 \text{ kPa}/^\circ\text{C}$ , respectively. While the combination 179/4.5 presents a slight improvement of the fit, the combination 131/3 still results in a reasonable fit with the possible exception of phases 1a and 1b where stresses are underpredicted significantly for cells 3-top and 4 (not shown). One may argue that the amount of insight is limited that can be gained from curve fitting alone, and we do not recommend an optimal set of parameters. Values in the range of  $n = 1$  to  $n = 4$  have been reported (e.g., Drouin and Michel, 1974; Duval et al., 2010; Song et al., 2006). However, one can gain insight from an analysis of temperature dependence of the assumed physical properties.

The performance of model Eqs. (5) to (8) will be compared next to identify the best compromise between number of parameters and quality of fit to observations. Using model Eqs. (6) to (8), the following observations were made (not shown): fitted results based on (6) are very similar to those of (5). The peaks in phases 1b and 2a of cells 3-top, 4, and 5-top are better predicted by (6) while the discrepancy in phase 4b of cells 3-top and 4 is larger. The latter is of particular concern as it appears to indicate that the increase of the creep rate with temperature is underestimated by (6) (note that this assessment assumes that  $n = 3.7$  is a valid assumption for both equations). Using (7) instead of (5) further exaggerates the modeled peaks in phases 1b, 2a and at the transition from phase 3 to 4a of cells 3-top, and 4 (not shown). Hence, we have no evidence that accounting for a temperature dependence of the elastic modulus leads to practical improvements of the model. It appears that more data both at low ice temperatures (i.e.,  $< -10^\circ\text{C}$ ) and close to the melting point of ice (i.e.,  $> -1^\circ\text{C}$ ) would be required to assess this further. Using temperature-independent ice properties in (8), some features are fitted slightly better while other features are

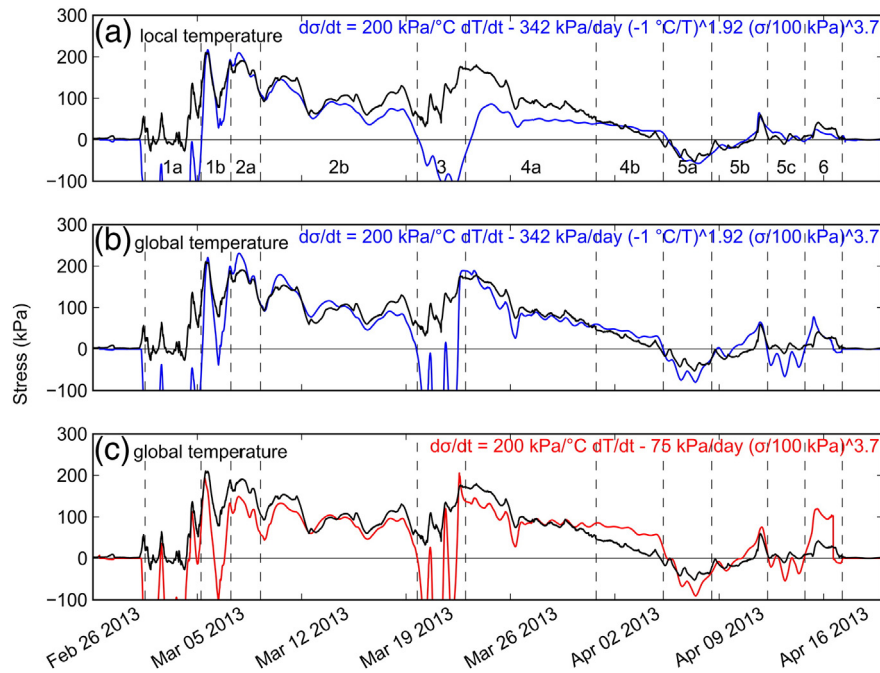


**Fig. 9.** Comparison of data from cell 4 data (black line) modeled with (a) temperature measured at cell 4, and (b) representative ice temperature characteristic for the ice sheet at large, and (c) with representative ice temperature and temperature-independent creep. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

fitted slightly worse than with (5) (Fig. 9c). The fit to cell 2 improves in every phase (not shown). However, the most striking discrepancy appears in phase 4b with stresses systematically overpredicted for cells 3-top, 4, and 5-top (Fig. 9c). Phase 4b was marked by ice temperatures above  $-1^{\circ}\text{C}$ , suggesting that creep rate increase with temperature is an

important feature for the range of ice temperatures observed in this study. While this conclusion is not new, there seem to be few examples of this effect in the literature. We conclude that Eq. (5) captures the most significant processes required to reproduce thermal loads of our measurement: elasticity and thermal expansion of the ice cover, creep

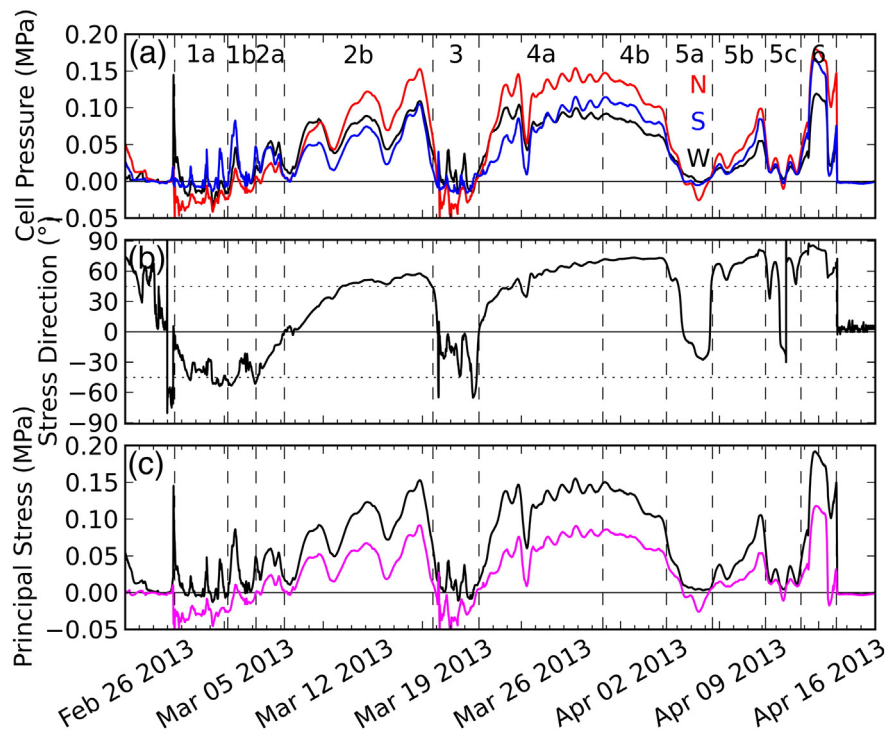


**Fig. 10.** Principal stresses derived from cells N, S, and W-top. (a) Data of cells N (red), S (blue), and W-top (black), (b) direction of the first principal stress, (c) magnitude of first (black line) and second (purple line) principal stresses. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

relaxation, and temperature-dependence of the creep rate. However, in the light of spatial variability of  $B$ , other effects may need to be included.

#### 4.3. Principal stresses

A simple numerical model of the stress field was used to verify the hypothesis that the rotation of the principal stresses is real and could have been caused by uneven ice temperature change without the need to invoke cracks. Although ice experiences creep, a simple thermoelastic model was used to study potential stress distribution qualitatively. Using the TNO DIANA™ software package, numerical simulations were performed of a square, thermoelastic plate of size  $30\text{ m} \times 30\text{ m}$  ( $\alpha = 5 \times 10^{-5}\text{ K}^{-1}$ ,  $E = 5\text{ GPa}$ ) with boundaries fixed at three sides, representing a simplification of Taraldsvikfossen Reservoir (Fig. 11). By design, the material was stress-free at  $-3\text{ °C}$ . The initial temperature of the material was  $0\text{ °C}$ , i.e. the material experienced compressive stress throughout the entire domain. The stress field was calculated while the temperature of the upper-left hand side quadrant was reduced to  $-4\text{ °C}$ . The development of the stress field is shown in Fig. 11. Initially, the material was under compressive stress and the first principal stress at the marked location was aligned approximately parallel to the direction of confinement (Fig. 11a). As the material warmed in the upper left hand side quadrant (Fig. 11b, temperature in the quadrant was  $-2.3\text{ °C}$ ), reducing the stresses, the direction of the principal stress changed clockwise at the location marked below. Eventually, the material in the upper left hand side quadrant experienced tensile stress (Fig. 11c, temperature  $-4\text{ °C}$ ), imposing essentially no compressive forces on the remaining material in the domain. The whole process resulted in a rotation of the principal stresses by approximately  $90^\circ$  in the quadrant below (e.g. point circled in Fig. 11a to c). The direction of the first principal stress circled in Fig. 11 is shown in Fig. 12 as a function of temperature in the upper left and side quadrant, illustrating that the transition was continuous but non-linear.

The simple numerical model cannot describe the stress field in the ice cover of Taraldsvikfossen Reservoir accurately. For example, the shape of the domain and boundary conditions are not accurate, both temperature distribution and temperature evolution are oversimplified, material properties are not temperature-dependent, creep of the material is ignored and cracks are not taken into account. However, it illustrates that a significant rotation of the principal stresses may well be due to differential temperature development in the ice and would be related to measurement position with respect to the boundary. Hence, biaxial measurements in conjunction with a numerical model could help specify the boundary conditions of a small reservoir.

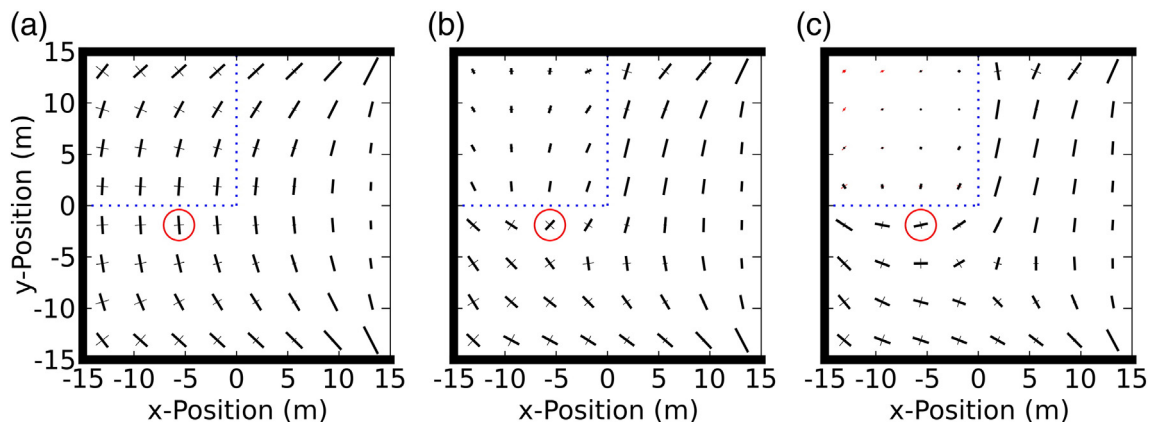


**Fig. 11.** Direction of the principal stresses in a thermoelastic materials confined at the upper, left, and lower boundary (first principal stress: thick lines, second principal stress: thin lines). Length of stress lines is proportional to stress magnitude. The angle of the principle stress at the point circled is shown in Fig. 12. (a) Homogeneous temperature throughout the domain, resulting in compressive stresses; (b) reduced temperature in upper left hand side quadrant (separated by dotted line), resulting in lower stress magnitudes and rotation of principal stress directions; (c) further reduced temperature in upper left hand side quadrant, resulting in tension (short red lines in upper left hand corner) and further rotation of principal stress directions.

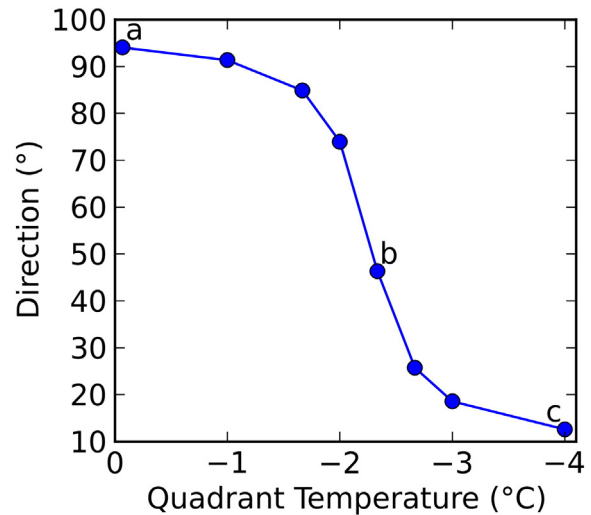


**Fig. 12.** Direction of the first principal stress (thick lines in Fig. 11) versus temperature of upper left hand side quadrant of the domain. The temperature of the remaining domain is  $0\text{ °C}$ . Data shown are of point  $x = -5.6\text{ m}$ ,  $y = -1.9\text{ m}$  in Fig. 11, markers a to c correspond to subplots of Fig. 11.

#### 5. Conclusions

In this study, spatially distributed ice stresses were measured in a small reservoir over the course of several weeks and the simple model of Bergdahl (1978) was used to describe thermal stresses in the ice cover. The agreement between model and observations was surprisingly good. The measurements showed that an uneven snow cover led to a spatially and temporally varying stress field. Comparing measurements and model, evidence of opening and closing cracks was found. The cracks did not seem to adversely affect the ability to model stresses normal to the boundary except that the magnitude of stresses in tension was overpredicted.

It was found that the temperature dependence of the creep rate is an important aspect to consider if ice temperatures range from  $-9\text{ °C}$  to temperatures close to the melting point. The approach of Cox (1984) worked well in this study even though it was based on a small set of data. There is potential for further improvement based on better knowledge of temperature-dependent ice properties.

One parameter used in the Bergdahl (1978) ice load model was found to vary systematically with location (i.e., parameter  $B$ ) with the lowest value observed at the center of the dam. This implies that actual

ice rheology can be difficult to determine from measurements in small reservoirs. Since the stress field should be homogeneous in a completely confined ice sheet, the spatial dependence of stresses registered in the ice and the systematic rotation of the principal stresses both point toward the absence of confinement at some section of the boundary. However, the systematic dependence of model parameters on location suggests that it should be possible to model spatial and temporal variability of stresses in small reservoirs numerically. In particular, earlier work found encouraging agreement between numerical models and field and laboratory measurements (Azarnejad and Hruday, 1998). Numerical models using intrinsic material properties and appropriate boundary conditions may be used to estimate effective parameters of models such as Eq. (5). In this context one may also wish to consider the structure's ability to deform (CFBR, 2013). In order to gain experience in defining numerical boundary conditions, biaxial measurements should be performed in reservoirs. A better understanding of the stress distribution will help in experiment design. With computational resources available today, a better understanding of thermal ice loads is within reach. However, more data on small reservoirs without waterlevel fluctuations may be needed.

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